## Exercise VI

1. Give three examples of (i) a sequence of the type $\left\{r^{n}\right\}$; (ii) a geometric series; (iii) an infinite decimal which converges to a rational number; (iv) a decreasing convergent sequence; (v) a decreasing non convergent sequence (vi) a sequence which converges to an irrational number.

Do not use any example that we have met already.
2. Sketch the graph of a function $f(x)$ with domain $[1,5], f(1)<0<f(5)$ and $f(x) \neq 0$ for any $x \in[1,5]$.
3. Use the Intermediate Value Theorem to prove that $x^{3}-3 x^{2}+10 x-7=0$ for some $x \in[0,1]$.
4. (a) Let $f(x)=x^{3}$. Is $f$ continuous over (i) $(-2,2)$ ? (ii) $[-2,2]$ ?
(b) Let $f(x)= \begin{cases}x^{3}, & x \in(-2,2) \\ -4, & x=-2 \\ 4, & x=2 .\end{cases}$

Sketch the graph of $f$.
Is $f$ continous over (i) $(-2,2)$ ? (ii) $[-2,2]$ ?
(c) Let $f(x)= \begin{cases}-3, & x \leq 1 \\ 3, & x>1 .\end{cases}$

Sketch the graph of $f$.
Is $f$ continous over (i) $(-2,2)$ ? (ii) $[-2,2]$ ? (iii) $[-1,1]$ ? (iv) $[1,2]$ ? (v) $(1,2]$ ?

