Exercise VI

1. Give three examples of (i) a sequence of the type $\{r^n\}$; (ii) a geometric series; (iii) an infinite decimal which converges to a rational number; (iv) a decreasing convergent sequence; (v) a decreasing non convergent sequence (vi) a sequence which converges to an irrational number.

Do not use any example that we have met already.

- 2. Sketch the graph of a function f(x) with domain [1,5], f(1) < 0 < f(5)and $f(x) \neq 0$ for any $x \in [1, 5]$.
- 3. Use the Intermediate Value Theorem to prove that $x^3 3x^2 + 10x 7 = 0$ for some $x \in [0, 1]$.
- 4. (a) Let $f(x) = x^3$. Is f continuous over (i) (-2, 2)? (ii) [-2, 2]?

(b) Let
$$f(x) = \begin{cases} x^3, & x \in (-2,2) \\ -4, & x = -2 \\ 4, & x = 2. \end{cases}$$

Sketch the graph of f. Is f continuous over (i) (-2, 2)? (ii) [-2, 2]?

(c) Let
$$f(x) = \begin{cases} -3, & x \le 1\\ 3, & x > 1. \end{cases}$$

Sketch the graph of f .
Is f continous over (i) $(-2, 2)$? (ii) $[-2, 2]$? (iii) $[-1, 1]$? (iv) $[1, 2]$?
(v) $(1, 2]$?